

Design of Multivariable PI Controller for Distillation Column using Simulated Annealing Algorithm

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Abstract

The main aim of this paper is to design a centralized PI controller for the bench mark distillation column process using Simulated Annealing. In general, multivariable system can be controlled by decentralized controller. The improvement of each single loop enhances the multivariable control. But multiloop controller fails when the interaction between loops are very high. In the presented method centralised PI controller is designed for highly nonlinear interacting process. In this paper, multivariable PI controller is designed and controller parameters are tuned using Simulated Annealing. The simulation results are given for the comparison of multivariable and multiloop PI controller. The superiority of proposed multivariable PI controller performance is briefly discussed.

Keywords - Multi-Input Multi-Output (MIMO), Distillation Column, Davison Method, Centralized Control, Simulated Annealing

1. INTRODUCTION

Most manufacturing and engineering processes are multi input multi output (MIMO) systems which have direct and indirect influence of input on the output. In any system, the manipulated input can be adjusted such way to regulate and track the required output. The coupled multivariable system has interaction effect which act as disturbance to the output. This inherited characteristics of the system cannot be changed, the only possible way to make a desired system is to design a closed loop control system. The control strategy for coupled multi input system is not easy like single independent loop system. The complicated multiloop single loop system has to be tuned such way to meet all the requirements of faster servo tracking and disturbance rejection. The performance enhancement of each single loop PID control provides improved performance in multivariable systems [1, 2]. Process interaction plays a very important function in scheming a fine controller, small intensity interaction is cancelled out by decoupler. It is an active component which is used to reduce interaction of control effort from one loop to another loop [3][4][5]. In real time, decoupler is not successful when the coupling effect among controlled loops is high because it cannot terminate the interaction effect entirely; therefore, there will be some leakage interaction present in the loops.

The design procedure of multiloop controller scheme involves interaction analysis, controlled loop pairing and detuning method for selection of controller parameters. Even detuning method works better for

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and bottom compositions of methanol respectively. The input u_1 and u_2 are the reflux rate and stream flow of the reboiler.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{-2.2e^{-t}}{7s+1} & \frac{1.3e^{-1.3t}}{7s+1} \\ \frac{-2.8e^{-1.8t}}{9.5s+1} & \frac{4.3e^{-0.36t}}{9.2s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (1)$$

III. MULTILoop AND MULTIVARIABLE PI CONTROLLER DESIGN

In manufacturing/process and engineering control problems, the system has multiple coupled inputs. It is difficult to control the required output product quality without compromising the outputs. The benchmark Two input and Two output system shown in the figure 3. The $G_{11}(s)$, $G_{12}(s)$, $G_{21}(s)$ and $G_{22}(s)$ are the process transfer function model. The centralized PI controller contains four different PI controllers to control the two outputs. The $g_{c11}(s)$ and $g_{c22}(s)$ are the controller which directly controls the desired output. The controller $g_{c11}(s)$ and $g_{c11}(s)$ are indirectly acting to control the outputs and directly reduce the interaction effect between each loop. The centralized PI controller structure is shown in figure 3. The controller seems like multiloop controller because of the 4 PI controller connections. But, this is a centralized controller which get two inputs and provides two controller input, hence each control inputs is the function of each control loop error and complete output of the system.

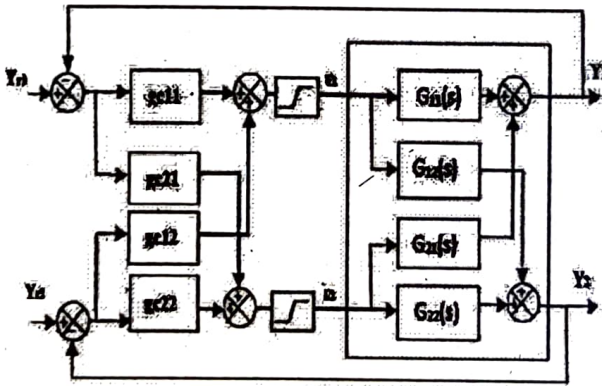


Fig. 3. Centralized PI Control Scheme

A. Davison Method

Davison has projected an empirical method of tuning a multivariable PI control system where it makes use of only the steady state gain matrix of the system for the design of centralized PI controllers. The steady state gain matrix of the multivariable stable system can be obtained from transfer function matrix. The proportional gain K_c and integral gain K_i of centralized PI controller is given in equ.2, 3.

$$g_{c21}(s) = \left(k_{c21} + \frac{k_{i21}}{s} \right)$$

$$g_{c12}(s) = \left(k_{c12} + \frac{k_{i12}}{s} \right)$$

$$g_{c22}(s) = \left(k_{c22} + \frac{k_{i22}}{s} \right)$$

IV. SIMULATION STUDY

The controller performance was analyzed by developing closed loop simulation model. The Centralized PI controller is designed using Davidson method and controller parameters are tuned using simulated annealing. The Centralized PI controller controller diagonal elements g_{c12} and g_{c21} controller minimize the interaction effect between two loops compared to multiloop ZN based PI controller. The improvement of each PI controller enhances the centralized PI controller. The tuning parameter is δ and ζ tuned by Simulated Annealing swarm optimization technique.

Table 1. The ZN PI controller settings

	ZN-PI $k_p=(0.9*t)/(k*d)$ $t_i=d/0.3$		
	Kp	ti	ITAE
Loop1	-2.8636	3.333	236.3
Loop2	5.348	1.2	

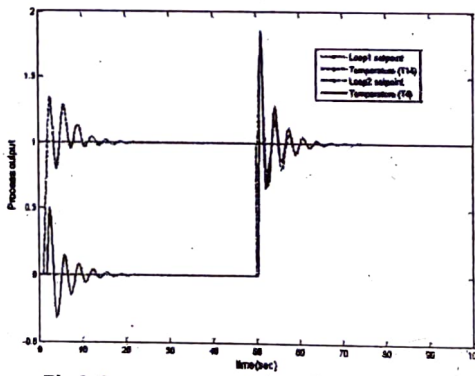


Fig.5. Servo response of Centralized PI controller

The multiloop ZN based PI controller performance is shown in figure 4. The setpoint changes are introduced as shown in figure 4, where the loop 1 temperature (T14) is maintained at 1 and the step variation is applied at 50 sec for loop 2 temperature (T2). At 50 sec, the loop 1 PI controller tries to track setpoint and loop 1 PI controller tries to regulate the disturbance due to the interaction effect. From the figure 4 & 5, it can be clearly seen that centralized PI controller reaches the setpoint smoothly with minimum settling time. The disturbance rejection of centralized PI controller is superior than the ZN based multiloop Controller.

$$F = \int_0^T [w_1 \cdot t |e_1(t)| + w_2 \cdot t |e_2(t)|] dt \quad (4)$$

$$F = 1/(1+0.5 \text{ ITAE (Loop1)} + 0.5 \cdot \text{ITAE (loop2)}) \quad (5)$$

C. Controller Design

The process with two input two output represented by

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \quad (6)$$

The steady state gain matrix of multivariable stable system is computed for designing centralized PI controller.

$$G_p(0) = K_p = \begin{bmatrix} -2.2 & 1.3 \\ -2.8 & 4.3 \end{bmatrix} \quad (7)$$

After the computation of the steady state gain matrix for selected region, the inverse form of gain matrix is

$$K_p^{-1}(s) = \begin{bmatrix} -2.2 & 1.3 \\ -2.8 & 4.3 \end{bmatrix}^{-1} \quad (8)$$

$$G_p^{-1}(0) = \begin{bmatrix} -0.7388 & 0.2234 \\ -0.4811 & 0.3780 \end{bmatrix} \quad (9)$$

The proportional and integral gain of centralized PI controller is given in equation 25, 26.

$$K_c(s) = \begin{bmatrix} -0.7388 \cdot \delta & 0.2234 \cdot \delta \\ -0.4811 \cdot \delta & 0.3780 \cdot \delta \end{bmatrix} \quad (10)$$

$$K_i(s) = \begin{bmatrix} -0.7388 \cdot \varepsilon & 0.2234 \cdot \varepsilon \\ -0.4811 \cdot \varepsilon & 0.3780 \cdot \varepsilon \end{bmatrix} \quad (11)$$

The multivariable controller

$$G_c(s) = \begin{bmatrix} g_{c11}(s) & g_{c12}(s) \\ g_{c21}(s) & g_{c22}(s) \end{bmatrix} \quad (12)$$

Where,

$$g_{c11}(s) = \left(k_{c11} + \frac{k_{i11}}{s} \right)$$

$$K_c = \delta [G_p(s=0)]^{-1} \quad (2)$$

$$K_i = \varepsilon [G_p(s=0)]^{-1} \quad (3)$$

Here $[G_p(s=0)]^{-1}$ is called the rough tuning parameters. The inverse of steady-state gain matrix is called as the rough tuning matrix.

IV. Controller Tuning using Simulated Annealing Algorithm

A. Simulated Annealing

The Simulated annealing algorithm is inspired from the process of physical annealing with solids. The crystalline solid is heated and allowed to cool very slowly to regular possible crystal configuration with superior structure integrity. This thermo dynamical characteristic is converted into optimization algorithm to solve engineering problems.

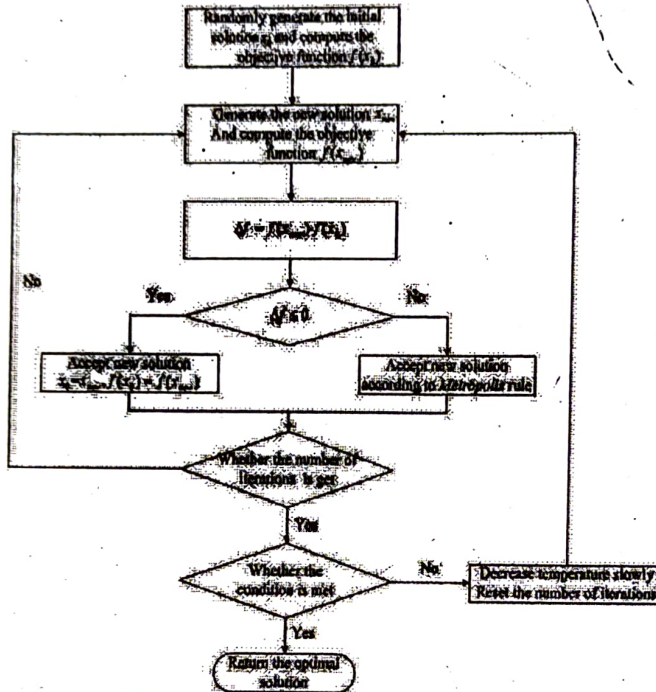


Figure 4. Simulated Annealing Algorithm [19]

B. Optimal tuning of the parameter δ and ε using Simulated Annealing Technique

The proportional gain K_c and the integral gain K_i of the centralized PI controller represented in equation 17 and 18 involves two tunable parameters δ and ε which ranges from 0 to 1, recommended range is 0.1 - 0.3. In the present work, an attempt is made to tune the parameter using particle swarm optimization technique minimizing the ISE of the errors in the loops.

The multi-objective optimal function is selected as

The objective function for controller design problem is formulated and the constraint bounds are given below,

The Integral Time Absolute Error (ITAE) value for ZN based controller setting is 236.3 which is higher than the centralized PI controller ITAE 199.4.

Table 2. Centralized PI controller settings

	δ	ε	ITAE
Centralized PI	3	0.75	199.4

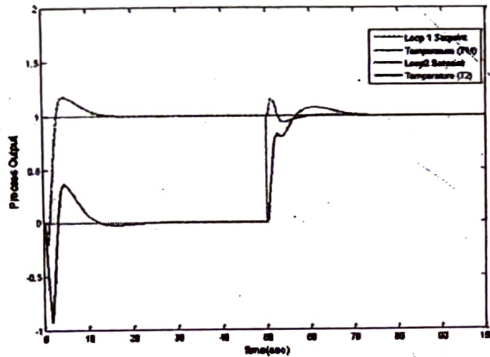


Fig. 6. Regulatory response of centralized PI controller

V. CONCLUSION

The multivariable PI controller is designed and then the tuning parameters are optimally determined by SA. The multivariable PI controller performance is improved through each PI controller. The proposed multivariable controller is capable of eliminating the interaction effect between loops by the additional diagonal controller. The controller parameter of the diagonal controller is in the negative gains which take care of eliminating the interaction effect by compensating the controlled input. Simulation studies show the superiority of SA based multivariable PI controller over ZN based multiloop ZN based PI controller.

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the process which has modest interaction between input and outputs. For the process with strong interaction system, detuning methods fails. The performance of detuning method is far away from optimal solution. In multiloop control schemes multiloop PID/PI controller is tuned by Sequential closing methods [6], independent design methods [7],[8], relay enhanced tuning [9], and internal model control (IMC) tuning [10]. This tuning method works well when system interaction is modest. A centralized controller gives desirable for severe interaction systems [11-15].

In this work, centralized $n \times n$ PI controller is used for distillation column control. The multivariable PI controller is designed based on the Davison method [11], and then the multivariable controllers are tuned using simulated annealing technique. This paper organized as follows, section II presents the description of the distillation column process. The design of multiloop PI and Multivariable PI controller scheme is discussed in section III. The analysis of closed loop controller performance is discussed with Simulation results in the section IV. The final conclusion of the paper is concluded in section V.

II. DISTILLATION COLUMN PROCESS DESCRIPTION

Distillation is one of the most significant unit activities in chemical engineering. The point of a distillation section is to isolate a mixer of components into at least two results of various compositions. The physical standard of partition in distillation is the distinction in the instability of the segments. The partition happens in a vertical section where warmth is added to a reboiler at the base and expelled from condenser at the top. A flood of fume delivered in the reboiler ascends through the section and is constrained into contact with a fluid stream from the condenser streaming downwards in the segment. The volatile parts are enhanced in the vapour stage and the less unpredictable (overwhelming) segments are advanced in the fluid stage. An item stream taken from the highest point of the section in this way for the most part contains light segments, while a stream taken from the base contains overwhelming segments.

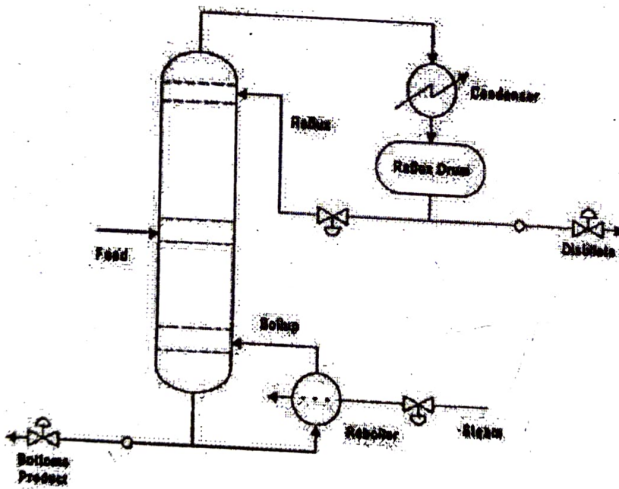


Fig. 1. Schematic diagram of Distillation column

The mathematical model of VL Column is given by Luyben (1986) is used in the paper as plant model. The two input two output model is shown in equation 1. The output y_1 and y_2 are the overhead